

# Accumulating evidence for nonstandard leptonic decays of $D_s$ mesons

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The measured rate for  $D_s^+ \rightarrow \ell^+ \nu$  decays, where  $\ell$  is a muon or tau, is larger than the standard model prediction at the  $3.8\sigma$  level. We discuss how robust the theoretical prediction is, and we show that the discrepancy with experiment may be explained by a charged Higgs boson or a leptoquark.

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*Introduction.*—The pattern of flavor and  $CP$  violation of the standard model has been established by a wide range of experiments. This agreement, however, leaves room for new flavor effects to show up as calculations and measurements improve. Intriguingly, decays of the  $D_s$  meson, which is the lightest  $c\bar{s}$  state, could be more sensitive to new physics than any other process explored so far. It suffices that a new particle couples predominantly to leptons and up-type quarks, but not to the first generation.

In this Letter we examine the leptonic decays of the  $D_s$ . Recently, the calculation of the relevant QCD matrix element has improved significantly, and more accurate measurements of the rate have been made. The average of the experimental results disagrees with the standard model by almost four standard deviations. We discuss the evidence and several explanations, including the possibility that a nonstandard amplitude interferes with the standard  $W$ -mediated amplitude.

*Leptonic  $D_s$  decays.*—The  $D_s \rightarrow \ell \nu$  branching fraction, where  $\ell$  is a charged lepton of mass  $m_\ell$ , is given in the standard model by

$$B(D_s \rightarrow \ell \nu) = \frac{m_{D_s}}{8\pi} \tau_{D_s} f_{D_s}^2 |G_F V_{cs}^* m_\ell|^2 \left(1 - \frac{m_\ell^2}{m_{D_s}^2}\right)^2. \quad (1)$$

Here  $m_{D_s}$  and  $\tau_{D_s}$  are the mass and lifetime of the  $D_s$ ,  $G_F$  is the Fermi constant, and  $V_{cs}$  is a Cabibbo-Kobayashi-Maskawa (CKM) element. The decay constant  $f_{D_s}$  is defined by

$$\langle 0 | \bar{s} \gamma_\mu \gamma_5 c | D_s(p) \rangle = i f_{D_s} p_\mu, \quad (2)$$

where  $p_\mu$  is the 4-momentum of the  $D_s$  meson. Although the electroweak transition proceeds at the tree level,  $D_s^+ \rightarrow W^+ \rightarrow \ell^+ \nu_\ell$ , its rate is suppressed. The helicity of the lepton must flip, leading to the factor  $m_\ell$  in the amplitude. For the muon, this helicity suppression  $(m_\ell/m_{D_s})^2$  is  $2.8 \times 10^{-3}$ . The  $\tau$  mass is only 10% smaller than the  $D_s$  mass (1.969 GeV), so there is no significant helicity suppression, but the phase space suppression [the last factor in Eq. (1)] is  $3.4 \times 10^{-2}$ .

We have collected in Table I all precise experimental measurements of  $B(D_s \rightarrow \ell \nu)$ , which are usually quoted

TABLE I: Experimental values of  $f_{D_s}$ . Our averages treat systematic uncertainties as uncorrelated and omit the PDG entry [1], which is an average of earlier experiments.

final state	reference	$f_{D_s}$ (MeV)
$\ell \nu$	PDG [1]	$294 \pm 27$
$\mu \nu$	BaBar [3]	$283 \pm 17 \pm 16$
$\mu \nu$	CLEO [4]	$264 \pm 15 \pm 7$
$\mu \nu$	Belle [5]	$275 \pm 16 \pm 12$
$\tau \nu$ ( $\tau \rightarrow \pi \nu$ )	CLEO [4]	$310 \pm 25 \pm 8$
$\tau \nu$ ( $\tau \rightarrow e \nu \bar{\nu}$ )	CLEO [6]	$273 \pm 16 \pm 8$
$\mu \nu$	our average	$273 \pm 11$
$\tau \nu$	our average	$285 \pm 15$

in terms of  $f_{D_s}$  [1, 2]. Combining the error bars in quadrature, our average of  $\tau \nu$  and  $\mu \nu$  final states is

$$(f_{D_s})_{\text{expt}} = 277 \pm 9 \text{ MeV}. \quad (3)$$

The most accurate calculation from lattice QCD is [7]

$$(f_{D_s})_{\text{QCD}} = 241 \pm 3 \text{ MeV}, \quad (4)$$

where statistical and systematic uncertainties are combined in the fitting methods. The only other modern lattice-QCD calculation agrees,  $249 \pm 3 \pm 16 \text{ MeV}$  [8], but its quoted error is five times larger and would not influence a weighted average with Eq. (4). The discrepancy between Eqs. (3) and (4) is 15% and  $3.8\sigma$ . Table I also shows averages for each mode separately: for  $\tau \nu$  ( $\mu \nu$ ) alone, the discrepancy is 18% and  $2.9\sigma$  (13% and  $2.7\sigma$ ).

If the BaBar result is omitted from the average, as in Ref. [2], then the discrepancy is  $3.4\sigma$ . On the other hand, if the earlier measurements [1] as well as the BaBar result are included, we find a  $4.1\sigma$  discrepancy.

*Experiments.*—CLEO [4, 6] produces  $D_s$  pairs not far above threshold, where the multiplicity is low. Their method reconstructs one  $D_s^{(*)}$  and then counts how often the opposite-side  $D_s$  decays leptonically. When the charged lepton is a muon, the neutrino is “detected” by requiring the missing mass-squared to peak at zero. When the charged lepton is a  $\tau$ , the identification is made

through the subsequent decays  $\tau \rightarrow e\nu\bar{\nu}$  and  $\tau \rightarrow \pi\bar{\nu}$ . BaBar [3] observes  $D_s$  mesons coming from the decay  $D_s^* \rightarrow D_s\gamma$ , produced well above threshold. They compare the relative number of subsequent  $D_s \rightarrow \mu^+\nu$  and  $D_s \rightarrow \phi\pi$ , and then use their own measurement of  $B(D_s \rightarrow \phi\pi)$  to determine  $B(D_s \rightarrow \ell\nu)$ . Belle [5] also observes  $D_s$  via  $D_s^* \rightarrow D_s\gamma$ , but the whole event is reconstructed, using a Monte Carlo technique. In summary, all these measurements have central values and error bars that are straightforward to interpret, and to combine to obtain Eq. (3).

The measured branching fraction and Eq. (1) yield  $|V_{cs}|f_{D_s}$ . The experiments appeal to three-generation CKM unitarity, either taking  $|V_{cs}|$  from a global fit to flavor physics [1], or setting  $|V_{cs}| = |V_{ud}|$ . The difference is numerically irrelevant. Relaxing the assumption cannot lead to agreement between theory and experiment because unitarity, even for more than three generations, requires  $|V_{cs}| < 1$ , whereas the discrepancy would require  $|V_{cs}| \approx 1.1$ .

*Radiative corrections.*—The measurements are not, strictly speaking, for  $D_s \rightarrow \ell\nu$  alone, because some photons are always radiated. The radiative corrections have been studied, focusing on effects that could overcome the helicity suppression [9, 10].

For  $D_s \rightarrow \tau^+\nu$  there is no sizable helicity suppression. In the rest frame of the  $D_s$ , the  $\tau$  acquires only 9.3 MeV of kinetic energy, so it cannot radiate much. This is borne out in the calculation [9], finding that the radiative corrections are genuinely of order  $\alpha$  and, thus, too small to explain the discrepancy [11].

For  $D_s \rightarrow \mu^+\nu$  radiative corrections could play a role due to processes of the form  $D_s \rightarrow \gamma D_s^* \rightarrow \gamma\mu^+\nu$ , where  $D_s^*$  is a (virtual) vector or axial-vector meson. The transition  $D_s^* \rightarrow \mu^+\nu$  is not helicity-suppressed, so the factor  $\alpha$  for radiation is compensated by a relative factor  $m_{D_s}^2/m_\mu^2$  for omitting helicity suppression. Using Eq. (12) of Ref. [9] and imposing the CLEO [4] cut  $E_\gamma > 300$  MeV, we find that the radiative rate is around 1% and, hence, insufficient to explain the discrepancy.

*Lattice QCD*—There are many lattice-QCD calculations for  $f_{D_s}$  in the literature, but only Refs. [7, 8] include 2+1 flavors of sea quarks, which is necessary to find agreement for many “gold-plated” quantities, namely those for which errors are easiest to control [12]. Both calculations start with lattice gauge fields generated by the MILC Collaboration [13], which employ “rooted staggered fermions” for the sea quarks. At finite lattice spacing this approach has small violations of unitarity and locality. Theoretical and numerical evidence suggests that these vanish in the continuum limit, such that QCD is obtained, with the undesirable features controlled with chiral perturbation theory. The strengths and weaknesses of this approach have been reviewed in detail [14].

Reference [7] reports an error five times smaller than that of Ref. [8] for several reasons. The largest uncertainties in Ref. [8] come from a power-counting estimate of the discretization error for the charm quark, and from uncertainties in the chiral extrapolation. Reference [7] employs a different discretization for the charm quark, which allows a controlled extrapolation to the continuum limit. Thus, the discretization error here is driven by the underlying numerical data.

The action for the charm quark in Ref. [7], called HISQ [15], is the same as that used for the light valence quarks. As a result the statistical errors are smaller than those of the heavy-quark method used in Ref. [8], and the axial current automatically has the physical normalization. The suitability of HISQ for charm is one of its design features, it has been tested via the charmonium spectrum [15], and the computed  $D$  and  $D_s$  masses agree with experiment. The  $D^+$  decay constant  $f_{D^+}$  also agrees with experiment, at  $1\sigma$ .

Another feature of Ref. [7] is the way the lattice-spacing and sea-quark mass dependence is fitted. Full details are not yet published, but it is noteworthy that the same analysis yields  $f_\pi$  and  $f_K$  in agreement with experiment [1] and earlier, equally precise, lattice-QCD calculations [16]. The  $D_s$  meson is simpler than the pion or kaon for lattice QCD, because none of the valence quarks is light, so  $f_{D_s}$  is easier to determine than  $f_\pi$ . Finally, simple extrapolations lead to the same central values for both  $m_{D_s}$  and  $f_{D_s}$ .

The result shown in Eq. (4) appears, therefore, to be a solid consequence of QCD. Similarly precise computations of  $f_{D_s}$  with other methods for lattice fermions should be carried out, but we do not see a simple reason to expect a substantial shift. Furthermore, even if the error bar were doubled, the discrepancy would remain significant:  $2.7\sigma$ ,  $2.5\sigma$ , and  $3.3\sigma$  for  $\tau$ ,  $\mu$ , and combined.

*Nonstandard effective interactions.*—Although the experiments quote the final states as  $\mu^+\nu_\mu$  and  $\tau^+\nu_\tau$  (and their charge conjugates), the flavor of the neutrino is not detected. Nonstandard physics could lead to any neutrino flavor, even a sterile neutrino. However, given the large effect that needs to be explained, we shall restrict our attention to amplitudes that could interfere with the standard model, which fixes the neutrino flavor. Lorentz-invariant new physics may contribute to  $D_s \rightarrow \ell\nu_\ell$  only through the following effective Lagrangian:

$$\frac{C_A^\ell}{M^2} (\bar{s}\gamma_\mu\gamma_5 c) (\bar{\nu}_L\gamma^\mu\ell_L) + \frac{C_P^\ell}{M^2} (\bar{s}\gamma_5 c) (\bar{\nu}_L\ell_R) + \text{H.c.}, \quad (5)$$

where  $C_A^\ell$  and  $C_P^\ell$  are complex dimensionless parameters,  $M$  is the mass of some particle whose exchange induces the four-fermion operators (5), and the  $c, s, \ell$  fields are taken in the mass-eigenstate basis.

The hadronic matrix element required for the decay induced by  $(\bar{s}\gamma_5 c)(\bar{\nu}_L\ell_R)$  is related to the one of Eq. (2)

by partial conservation of the axial current:

$$\langle 0 | \bar{s} \gamma_5 c | D_s \rangle = -i f_{D_s} m_{D_s}^2 (m_c + m_s)^{-1}. \quad (6)$$

The branching fraction in the presence of the operators (5) is given by Eq. (1) with  $G_F V_{cs}^* m_\ell$  replaced by

$$G_F V_{cs}^* m_\ell + \frac{1}{\sqrt{2} M^2} \left( C_A^\ell m_\ell + \frac{C_P^\ell m_{D_s}^2}{m_c + m_s} \right), \quad (7)$$

with no helicity suppression in the last term.

The imaginary part of  $V_{cs}$  is negligible (in the standard CKM parametrization [1]), so constructive interference, which would increase  $B(D_s \rightarrow \ell \nu)$ , requires the real part of  $C_A^\ell$  or  $C_P^\ell$  to be nonzero and positive. Assuming only one nonzero coefficient, the amplitude for  $\tau^+ \nu_\tau$  ( $\mu^+ \nu_\mu$ ) could be increased by 12% (8.4%) only if

$$\frac{M}{(\text{Re } C_A^\ell)^{1/2}} \lesssim \begin{cases} 710 \text{ GeV for } \ell = \tau \\ 850 \text{ GeV for } \ell = \mu \end{cases}, \quad (8)$$

$$\frac{M}{(\text{Re } C_P^\ell)^{1/2}} \lesssim \begin{cases} 920 \text{ GeV for } \ell = \tau \\ 4500 \text{ GeV for } \ell = \mu \end{cases}, \quad (9)$$

thereby reducing the discrepancy to  $1\sigma$  in each case. These bounds are a key result, because they constrain any model of new physics.

The effective interaction (5) also contributes to the semileptonic decays  $D \rightarrow K \mu^+ \nu$ . This proceeds through two amplitudes, corresponding to angular momentum  $J = 1$  or  $0$  for the lepton pair. For  $J = 1$ , the standard-model amplitude and that from  $C_A^\mu$  are not helicity suppressed, while that from  $C_P^\mu$  is. For  $J = 0$ , the pattern of helicity suppression is as for the leptonic decay. Hence, only the  $J = 1$  part of the rate will be visible, and as the accuracy of the lattice-QCD calculations improves, the comparison with experiment will help decide which interactions are responsible for the effect in  $D_s \rightarrow \ell \nu$ . The current status favors  $C_P^\mu \neq 0$  rather than  $C_A^\mu \neq 0$ , because the lattice-QCD prediction for  $D \rightarrow K \mu \nu$  [17] agrees with experiment [18], albeit at the  $\sim 7\%$  level.

*New particles.*—There are three choices for the electric charge of a boson that can mediate the four-fermion operators (5):  $+1, +2/3, -1/3$ , corresponding to the three diagrams shown in Fig. 1. The exchanged boson (taken to be emitted from the vertex where  $c$  is absorbed) is a color singlet if the electric charge is  $+1$ , and a color triplet if the electric charge is  $+2/3$  or  $-1/3$ . We shall consider only the cases where the new boson has spin 0 or 1, and its interactions are renormalizable.

A new vector boson,  $W'$ , of electric charge  $+1$  would contribute only to  $C_A^\ell$ . Such a boson must be associated with a new gauge symmetry, which makes it difficult to allow large couplings to left-handed leptons. One possibility is that  $W$  and  $W'$  mix, but the constraint from electroweak data on mixing ( $\lesssim 10^{-2}$ ) is too strong to

allow noticeable deviations in  $D_s$  decays. Another possibility is that some new vector-like fermions transform under the new gauge symmetry and mix with the left-handed leptons. Such mixing is also tightly constrained, especially by the nonobservation of vector-like fermions at LEP and the Tevatron. Overall, a  $W'$  is inconsistent with Eq. (8), barring perhaps some finely-tuned elaborate model (*e.g.*, with large  $W$ - $W'$  mixing whose electroweak effects are cancelled by other particles).

A spin-0 particle of charge  $+1$ ,  $H^+$ , appears in models with two or more Higgs doublets. Its interactions, in the mass eigenstate basis for charged fermions, include

$$H^+ (y_c \bar{c}_R s_L + y_s \bar{c}_L s_R + y_\ell \bar{\nu}_\ell \ell) + \text{H.c.}, \quad (10)$$

where  $y_c, y_s, y_\ell$  are complex Yukawa couplings. The exchange of  $H^+$  induces  $C_A^\ell = 0$  and

$$C_P^\ell = \frac{1}{2} (y_c^* - y_s^*) y_\ell, \quad (11)$$

taking  $M$  equal to the  $H^+$  mass. If  $H^+$  is the charged Higgs boson present in the Type-II two-Higgs-doublet model, then  $y_c/y_s = m_c/(m_s \tan^2 \beta)$  so that  $C_P^\ell$  can have either sign [19], but the Yukawa couplings are too small to be compatible with Eq. (9).

Other models may lead to large constructive interference. For example, a two-Higgs-doublet model where one doublet gives the  $c, u$  (but not  $d, s, b$ , or  $t$ ) and lepton masses, and has a vacuum expectation value of about 2 GeV, yields  $|y_s| \ll y_\tau, y_c^* \sim O(1)$ . Thus,  $C_P^\ell > 0$  and the limits (9) are satisfied for  $M \lesssim 500$  GeV. Furthermore, such a model explains why the deviations in  $\tau \nu$  and  $\mu \nu$  are comparable. It is encouraging that this two-Higgs-doublet model does not induce tree-level flavor-changing neutral currents, and the off-diagonal couplings of  $H^+$  are CKM suppressed. Nevertheless, one-loop contributions to flavor-changing processes could require some fine tuning to evade experimental bounds.

The charge  $-1/3$  and  $+2/3$  exchanges correspond to leptoquarks. A scalar charge  $+2/3$  exchange arises for the  $(3, 2, +7/6)$  set of  $SU(3)_c \times SU(2)_W \times U(1)_Y$  charges. This leptoquark appears, for example, in a new theory of quark and lepton masses [20]. Let  $r = (r_u, r_d)$  be the doublet leptoquark, where  $r_d$  is its charge  $+2/3$  component. The interaction terms relevant here, written in the same basis as (5), are  $\lambda_{c\ell} r_d \bar{c}_R \nu_L^\ell + \lambda'_{s\ell} r_d \bar{s}_L \ell_R$ . The  $r_d$  exchange gives  $C_A^\ell = 0$  and  $C_P^\ell = -\lambda_{c\ell}^* \lambda'_{s\ell} / 4$ . Since the

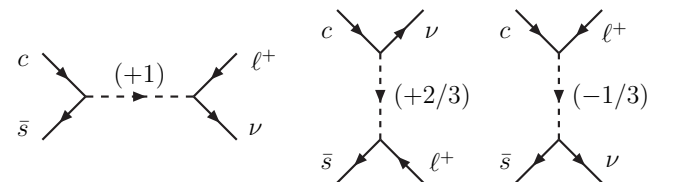


FIG. 1: Four-fermion operators induced by boson exchange.

leptoquark couplings can have any phase, the new amplitude can interfere constructively. Still, various flavor processes constrain the couplings of  $r$ . Even if its couplings to first-generation fermions were negligible, the lepton-flavor violating decays  $\tau \rightarrow \mu \bar{s} s$ , where  $\bar{s} s$  hadronizes to  $\eta$ ,  $\eta'$ ,  $\phi$  or  $K \bar{K}$ , set a lower limit on  $M^2/|\lambda'_{s\tau}\lambda'_{s\mu}|$ , which is hard to reconcile with Eq. (9). One way out would be a model with two  $r$  leptoquarks, with one coupling to  $\tau$  and the other one to  $\mu$ . The constraint from  $\tau \rightarrow \mu \bar{s} s$  similarly disfavors spin-1 leptoquarks of charge  $+2/3$ .

A scalar leptoquark of charge  $-1/3$  (also discussed in [20]) arises in the case of two sets of  $SU(3)_c \times SU(2)_W \times U(1)_Y$  charges:  $(3, 1, -1/3)$  or  $(3, 3, -1/3)$ . Let us denote the former by  $\tilde{d}$ . Its Yukawa couplings are given by

$$\tilde{d} [\bar{c}_L \ell_L^c - \bar{s}_L \nu_L^c] + \kappa'_\ell \bar{c}_R \ell_R^c + \text{H.c.}, \quad (12)$$

where  $\kappa_\ell$  and  $\kappa'_\ell$  are complex parameters. These interactions are present, for example, in  $R$ -parity violating supersymmetric models (their effect on  $D_s \rightarrow e^+ \nu$  has been analyzed in Ref. [21]). The  $\tilde{d}$  exchange, as in the last diagram of Fig. 1, gives (for  $M$  equal to the  $\tilde{d}$  mass)

$$C_A^\ell = \frac{1}{4} |\kappa_\ell|^2, \quad C_P^\ell = \frac{1}{4} \kappa_\ell \kappa'_\ell{}^*. \quad (13)$$

For  $|\kappa'_\ell/\kappa_\ell| \ll m_\ell m_c/m_{D_s}^2$ , the interference is automatically constructive [see Eq. (7)], and the resulting deviations in  $\tau \nu$  and  $\mu \nu$  are approximately equal. Moreover, there are no severe constraints from other processes on the couplings  $\kappa_\ell$  and  $\kappa'_\ell$  with  $\ell = \tau$  or  $\mu$ . The  $\tilde{d}$  couplings to the electron can be forbidden by a symmetry, and its couplings to first-generation quarks could be small.

The  $(3, 3, -1/3)$  scalar leptoquark includes an  $SU(2)_W$  component of charge  $-4/3$  which mediates  $\tau \rightarrow \mu \bar{s} s$ . The vector leptoquark of charge  $-1/3$  has the same problem.

*Conclusions.*—We have argued that the  $3.8\sigma$  discrepancy between the standard model and the combined experimental measurements of  $D_s \rightarrow \ell \nu$  appears so far to be robust, and thus it is worth interpreting it in terms of new physics. The upper bounds (8) and (9) on the scale of four-fermion operators are low enough to allow exploration of the underlying physics at the LHC.

A  $\tilde{d}$  scalar leptoquark of charge  $-1/3$  may solve the  $D_s$  puzzle without running into conflict with any other measurements. At the LHC, the  $\tilde{d}$  can be strongly produced in pairs, and the final states would be  $\ell^+ \ell^- j j$ , where  $\ell$  is a  $\tau$  or a  $\mu$ , and  $j$  is a  $c$ -jet. Given that there are two  $\ell j$  pairs, each of them forming a resonance at the  $\tilde{d}$  mass, the backgrounds can be kept under control.

An alternative explanation is provided by an  $H^+$  exchange in a model where a Higgs doublet gives masses to the charged leptons and  $c$  and  $u$  quarks, and a second Higgs doublet gives masses to the down-type and top quarks. Remarkably, both the leptoquark and charged Higgs solutions lead naturally to comparable increases in

the branching fractions for  $D_s \rightarrow \tau^+ \nu$  and  $D_s \rightarrow \mu^+ \nu$ , as suggested by the data.

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